Same Task, Different Paths: Catering for Student Diversity in the Mathematics Classroom

"For all teachers and all lessons most of the class should be working on tasks beyond their current levels of thinking" (Sullivan, 2008).

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describes how rich tasks can be adapted to cater for a diverse range of learners.

This statement poses a challenge for mathematics teachers in the classroom. Teachers do not want to have students simply marking time by completing tasks from which there is no new learning for them. Similarly, teachers do not want students struggling with tasks that are demoralisingly difficult. In one primary mathematics classroom, however, a teacher can expect to have a wide range in the levels of understanding of the students, some say as much as a five year span. So how do teachers deal effectively with the diversity of mathematical needs they find in their classrooms?

Some teachers aim lessons at the middle group in their class, reasoning that this will reach the maximum number of students. Different tasks are typically set for those students who cannot complete this core task. Common practices include the teacher repeating explanations for students having difficulty, and/or remaining with these students for most of the lesson. On the other hand, students who quickly and easily complete the core task are often given more "work" on the same topic or directed to complete additional and sometimes unrelated tasks.

Another way teachers try to cater for their students is by having "focus groups" where groups of students with similar needs are taken by the teacher for a task often

different from the independent work that the remainder of the class is completing. The aim of such groups is to provide focussed teaching to meet this group's needs. This kind of model is similar to that found in literacy and sometimes also extends to teachers having many different tasks and groups running simultaneously in mathematics lessons. I propose that such strategies are onerous for teachers, increasing their workload as a result of the need to plan four or five times as many tasks per lesson. Furthermore, the worth of such approaches, in terms of student learning, may not justify such efforts. Students who are not part of the teacher focus group can be left to their own devices while students requiring harder thinking might only get more work of the same type. Students who struggle may simply get more "practice" on something they probably did not understand the first time. This fragmentation of the class group is also a concern for students' self-esteem. It leaves the class no point of commonality for the discussion and interaction that is vital to give the sense that the class are a community of learners.

What teachers need are tasks in which the whole class can engage and which are easily adjusted so they can be increased in complexity to extend understanding or simplified to scaffold student learning. One task, one class community, one mathematical concept and one sane teacher!

The importance of using rich mathematical tasks

Rich mathematical tasks have been described as tasks that:

- create opportunities for students to explore and articulate mathematical ideas independently (Olson & Barrett, 2004);
- prompt student thinking and discussion (Schwan Smith, 2001);
- build student capacity for mathematical thinking and reasoning (Stein, Grover & Henningsen, 1996);
- are problematic in that the students usually have no "ready-made" procedure to use to solve the task, prompting the creation of strategies, or the task causes students to confront misconceptions (Cobb, Wood, Yackel, Wheatley, McNeal & Preston, 1989);
- hold potential for supporting students' development of mathematical interests;
- hold potential for students to access important mathematical ideas (Hodge, Visnovska, Zhao & Cobb, 2007);
- must be accessible to everyone at the start; and
- need to allow further challenges and be extendible (Ahmed, 1987).

The final two statements raise the point that rich tasks should be easily varied to allow for extending understanding and for scaffolding emerging understanding to cater for a diversity of student needs.

Rich mathematical tasks have the ability to reach most children at the point where their known understandings meet the unknown. Vygotsky (1978) described this point as the "zone of proximal development." In this zone, the students understand some of what is needed to build new understandings, but not all. They are neither bored nor stressed. With support working in this zone, new understandings can be built while the student remains challenged, engaged and curious.

Of course in a mathematics classroom, there will be many different zones of proximal development because of the diversity in students' attainment levels, attitudes, past experiences and prior learning. One important key to designing tasks with variations is knowing where students are operating in terms of the mathematics of the task so as to provide tailoring of the task to match their needs.

This does not mean that there needs to be 25 different variations of a task. Generally students will develop in similar ways when understanding mathematical concepts. Teachers need knowledge of the common stages students move through when learning

the concept as well as common misconceptions — key components of pedagogical content knowledge in mathematics (Shulman, 1987). In this way, teachers can be ready to provide pre-planned variations of the task when students show that they require them.

Buses and children: A classroom example of a rich task and variations

This task was given to a Prep/Grade 1 composite class. As many teachers would know, the diversity of understanding and knowledge within a "straight" Prep grade can be overwhelming, let alone adding a group of Grade 1 children to the mix! These children represented a large span of mathematical development. Designing lessons in which the maximum number of children are working at their point of development is challenging. After some trial and error, I have found that the key is in choosing rich tasks, planning the variations the children might need, and drawing everyone together around the same concept.

The task I will describe here is "Buses and Children." Its primary focus is on place value ideas of tens and ones. The task aims to build understanding of ten as a unit and links to the notation of two-digit numbers. For other children, the task will build the development of counting a group of around 30 items with accuracy and the concept of "full" (tens) and "other ones" that are left over.

To begin, the children were posed this problem:

> The children in Prep/One and the Kinder children are going on an excursion to the Marine Discovery Centre. With our 19 children and the 14 Kinder children, there are 33 children altogether. The buses only hold 10 children at a time.
>
> **How many buses will we need? How many children will be on each bus?**

For children who have a grasp of tens and ones, of course, this task would not be problematic but this task is a problem for these children who are not yet operating with tens as units. In terms of where the children may have been starting from in their understanding of tens as a unit, I knew that:

- most of the Grade ones had just started to grapple with tens as a unit;
- a couple of Grade ones were on their way to understanding this concept more;
- most Preps would not have an idea about ten as a unit; and
- some Preps were still consolidating accurate counting of groups beyond 20.

In order to vary the task effectively, I followed these steps:

1. I discussed the context of the problem at length but not strategies to solve it. I showed the tens frame bus to give all the children a visual model of the buses (See Figure 1).

Figure 1. A tens frame bus.

2. I allowed all the children to make a start by thinking about the problem before offering variation. (This displays high expectations as sometimes the children can surprise you with what they can do on their own!)
3. I stepped in with lower level scaffolds, such as drawing the first bus for them when children were having trouble starting.
4. I provided the next stage of modelling for those who were consolidating counting stages such as one-to-one correspondence and counting a collection more than 20. A tens frame bus and materials used are shown in Figure 2.

Figure 2. Tens frame bus and materials.

5. I provided quick extensions on the same idea for those who showed an understanding of 10 as a unit by increasing the number of children. This means that drawing the buses is no longer efficient so more abstract reasoning is required. If this was successful, I offered a number over 100 to extend reasoning to 10 tens. One student's work on such a task is shown in Figure 3.

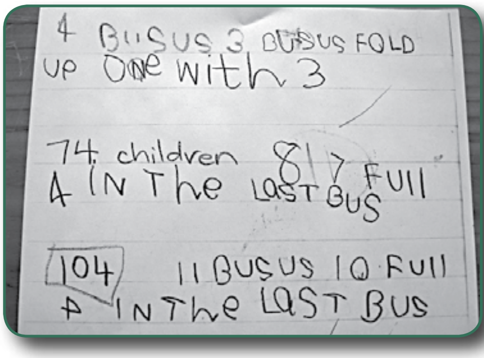

Figure 3. Student work with larger numbers.

At the conclusion of the task, I chose three children to share their solution strategies with the class. These children represented the levels of sophistication I had observed while the class worked on the task so one child showed modelling with ten frames buses and counters, another drew the buses and children and the third wrote a sentence in her book. The class talked about how all these strategies had resulted in the same answer, and how they were similar and different. In particular, the child who wrote the sentences explained how she knew that 33 was three full buses (tens) and three others "just by looking at the number". We explored this further by linking other two-digit numbers to the number of full buses (tens) and other ones and created a chart together (see Figure 4). By this stage some children who had only modelled the problem initially were beginning to say (loudly and excitedly) they could tell "just by looking at the number" how many full buses and other ones there would be. This was an important first step and although I too was excited, I knew that this concept required more consolidation in future lessons.

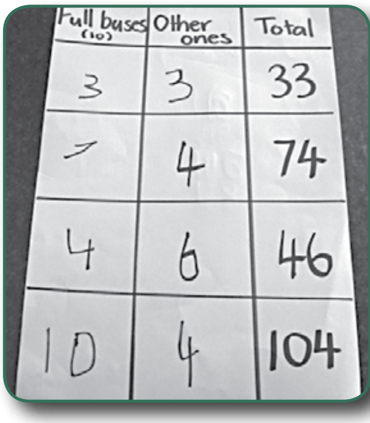

Figure 4. Making the mathematics explicit.

What made this task rich?

This task was rich in that it was easily "adjustable" to suit the various needs of the students in this class (Ahmed, 1987). Students who needed consolidation of counting collections could do so during this task and those who were beginning to understand 10 as a unit also had these understandings deepened through this task. Nevertheless, it was important that the same core concept could be discussed as a community of learners at the end of the lesson (Schwan Smith, 2001). In this way all the children had a common experience on which to reflect and that could be built upon in future lessons.

The use of visual representations and concrete materials also made this task rich. These aids can become mental models for the children to use when considering place value ideas such as tens and ones (Hodge et al., 2007). The materials were also easily varied. The tens frame bus was a common representation considered by all the children. A couple of children did not use the buses in their working but independently wrote sentences with numbers instead. Most children drew the buses and children, and a few also used counters to represent the problem. These variations were quick and easy to make but once again the tens frame bus became the common representation that all the children could discuss and on which they could all reflect.

The task was deceptively simple. All the children could independently explore the

concept of tens and ones through the task. The task was relevant to their real world experiences of going on buses to excursions and also allowed them to explore the important mathematical idea of ten as a unit.

Some other examples of rich tasks

I use rich tasks with all levels of the primary school in mathematics. I find that by using such tasks, I am (more or less) satisfied that the students have been working at the edge of their understanding, are engaged and challenged, and come together as a learning community around a common mathematical concept. Other examples of tasks I have found to be rich and easily varied are:

- The Arrays Game, suited mainly for middle primary grades, where players roll two dice and multiply these to colour an array on grid paper. Variations can be made by using different sided dice;
- Pentonimoes, making as many unique shapes as possible using five squares, then putting these shapes together to form a rectangle. This task, for middle to upper primary grades, encourages mental manipulation of the shapes; however, making models of the shapes offers scaffolding;
- Open-ended tasks such as "The answer is x; what could the question be?" or "How many ways can I make a tower of twelve unifix blocks using only two colours?" are suitable for all grade levels. Students requiring extension work can be asked to find a general rule or pattern related to the range of possible solutions.

In conclusion

The "Buses and Children" task was effective in that I prepared one task and planned for slight variations, the children were all working at their point of development, and a community of learners was maintained by keeping the task goals and core task the same for all the children.

Teachers need to source mathematically rich tasks that are easy to vary, know the range of ways in which their students may respond to these tasks, and plan for task variations to cater for those who require scaffolding and those who require extending. Rich tasks like "Buses and Children" make the difficult job of catering for diversity a little easier for teachers. More importantly, they also provide an opportunity for learning important mathematics for the children we teach.

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